

GEOELECTRICAL DATA BASED STATISTICAL MODELLING FOR YIELD OF BORE WELLS IN A TYPICAL KHONDALITIC TERRAIN

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ABSTRACT

The Kandivalasa river sub-basin a typical hard rock terrain covered with khondalitic (garnetiferous sillimanite gneiss) formation, situated near Vizayanagaram district of Andhra Pradesh, India is selected for the geoelectrical data based statistical modelling for determining the yield of borewells. The well yields are correlated to certain geoelectrical parameters obtained and derived from vertical electrical sounding data at 42 existing bore wells in the basin by using correlation and cluster analysis. It is observed that there is no correlation between the yield and any of the parameters. Cluster analysis has demonstrated that the high yielding cluster wells are characterised by an average value of transverse resistance of 1227 ohm-m² and an average value of aquifer resistance of 1093 ohm-m² with the aquifer resistivity ranging between 23 -43 ohm-m. and its thickness varying in the range of 27 -48 meters.

Introduction

Hardly anywhere on the earth is groundwater more difficult and assess than in the arid and semi-arid regions of hard rock terrain. Consequently location of high yielding wells in arid and semi-arid hard rock regions is a very difficult task. It is also in these regions that groundwater is most needed as surface water is limited due to erratic and minimal rainfall.

The Kandivalasa river sub-basin (Fig. 1) is one such semi-arid basin composed of a hard rock formation known as khondalitic (garnetiferous simmimanite gneiss) formation. It is located near Cheepurupalli town of Vizayanagaram District of Andhra Pradesh, India. The problem here is to locate the sites for irrigation bore wells yielding 8000 litres per hour (LPH) or more as has been stipulated by the government financing institutions to fund the Bore well irrigation schemes for small and marginal farmers. One more stipulation is that there should be 75% success rate with the above yield norm. Quite often it is observed that the successful well (Yield ≥ 8000 LPH) and unsuccessful well are located within tens of metres of distance in spite of the similar hydrogeological and geoelectrical characteristics, and has become difficult to achieve the desired success rate. To overcome this difficulty detailed and comprehensive examination of geoelectrical data at 42 drilled borewells was undertaken with a view to correlate them with the well yields by using correlation and cluster analysis.

Earlier Work

For a long time geophysicists have been seeking a solution to the problem of prediction of well yields before drilling (Way, 1942; enslin, 1955; Vincenz, 1968). Legrand (1967) has proposed a point value method for prediction of well yields in granitic terrain. Matveev et al (1974) proposed empirical relations between aquifer resistivity and yield of the well. Patangay et al (1977)

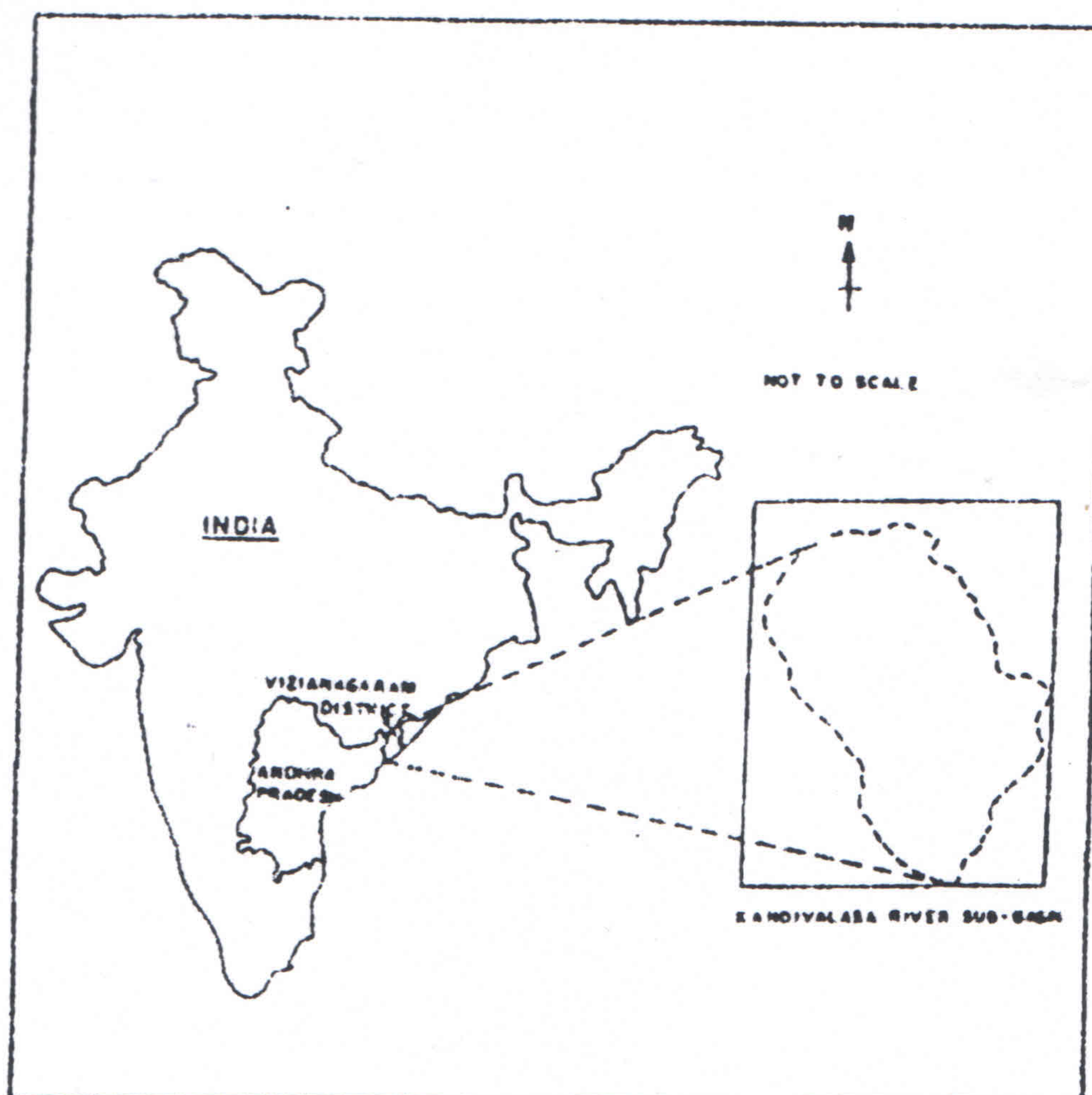


Fig. 1 - Location of Kandivalasa river sub-basin

correlated the resistivity of weathered granitic aquifer, well yield and coefficient of well penetration. Sporry et al (1991) correlated the transverse resistance of the aquifer to its Transmissivity and observed both positive and negative correlations between the parameters for various strata.

While the above studies were carried out in granitic terrain, no such work has been reported in the khondalitic terrain typical of the study area. In addition to the correlation analysis multivariate statistical analysis such as cluster analysis was also carried out in this paper.

Data Collection and Analysis

Over 80 vertical electrical soundings (VES) were conducted at hydrogeologically favourable locations covering entire basin. The data were interpreted with the help of an Iterative Computer Program. Except in one case which is a 2 layer model, at all other VES points, the subsurface system contains either 3 or 4 geoelectric layers. Depending on layer thicknesses and resistivities and also the local hydrogeological conditions, 42 wells were drilled out of which only 27 were successful as per the above yield criterion putting the success rate at only 64%. In order to improve the success rate and to establish a relation between geoelectrical data and well yield, eight geoelectrical parameters viz., (1) aquifer resistivity (ρ_1), ohm-m, (2) aquifer thickness (h), m, (3) depth to geoelectric basement (H), m, (4) Longitudinal conductance (L_c), mhos, (5) average longitudinal resistivity (ρ_L), ohm-m, (6) transverse resistance (T_r), ohm-m², (7) average transverse resistivity (ρ_r), ohm-m, (8) aquifer resistance (A_r), ohm-m², of the 42 wells and their yields were selected for the analysis. Yields mentioned here are the compressor yields measured with 90° V-notch at the time of drilling. While the first three geoelectric parameters were directly taken from the VES data of 42 wells, the remaining five parameters are derived from the same VES data. From each VES location it is possible to identify the water bearing formation or aquifer. For example, the second layer in the VES data shown in Table 3 is the aquifer. Since this layer plays an important role for the success of the well, its thickness (h) and resistivity (ρ_1) are taken as two among the eight geoelectric parameters selected for analysis. As depth to the basement (H) gives the total column of weathered and fractured thickness for

accumulation of water, it is also selected for analysis. The criteria for selection of the other five parameters are as follows.

In any mathematical analysis the joint influence of all the $2n$ geoelectric variables involved in a multiple layer sub-surface system consisting of n layers, can be condensed into the following two terms (Bhattacharya and Patra, 1968):

$$\text{Longitudinal Conductance : } L_c = \sum_{i=1}^n (h_i / \rho_i) \quad (1)$$

$$\text{Transverse resistance : } T_r = \sum_{i=1}^n (h_i * \rho_i) \quad (2)$$

The former represents the case of flow parallel to the layers and the latter signifies the flow across layers. Similar to the concept of transverse resistance, the term **AQUIFER RESISTANCE** (A_r) is introduced in this paper. The aquifer resistance is defined as the product of aquifer resistivity and aquifer thickness. Thus,

$$A_r = \rho_i * h \quad (3)$$

This term is introduced with a view to enhance the role of the aquifer in the cluster analysis to be discussed in the later section. The units of L_c are mhos and the units of T_r and A_r are ohm-m². L_c and T_r are frequently transformed into more familiar resistivity units ohm-m by averaging them over the total depth H as follows:

$$\text{Average longitudinal resistivity } (\rho_L) = H/L_c \quad (4)$$

$$\text{Average transverse resistivity } (\rho_T) = T_r/H \quad (5)$$

Values of all the variables (3 of measured type and 5 of derived type) at the 42 bore well sites together with the well yields are given in Table 1.

Table 1 - Matrix of Geoelectric Parameters used in Statistical Modelling

| Geo=> | Aquifer | Aquifer | Depth | Longi- | Avg. | Trans- | Avg. | | Yield |
|----------|--------------|---------|-------|-----------|--------------|--------------------|--------------|--------------------|--------|
| electric | Resis- | Thick- | to | tudinal | Longi- | verse | Trans- | Aquifer | of the |
| Para- | tivity, | ness, | Base- | Conduc- | dinal | Resis- | verse | Resist- | Well, |
| meters | ohm-m | m | ment, | tance, | Resis- | tance, | Resis- | ance, | Litres |
| | | | m | mhos | tivity, | ohm-m ² | tivity, | ohm-m ² | per |
| | | | m | ohm-m | ohm-m | ohm-m ² | ohm-m | ohm-m ² | Hour |
| S.No. | (ρ_t) | (h) | (H) | (L_c) | (ρ_L) | (T_r) | (ρ_T) | (A_r) | (Q) |
| 1 | 10.27 | 18.24 | 18.76 | 1.79926 | 10.427 | 198.97 | 10.606 | 187.32 | 510 |
| 2 | 27.78 | 29.90 | 32.57 | 1.11024 | 29.336 | 1040.75 | 31.954 | 830.62 | 4,550 |
| 3 | 19.49 | 23.03 | 24.94 | 1.20410 | 20.713 | 611.24 | 24.508 | 448.85 | 4,550 |
| 4 | 21.96 | 18.46 | 22.48 | 1.22933 | 18.286 | 486.64 | 21.648 | 405.38 | 3,640 |
| 5 | 33.06 | 27.10 | 29.22 | 0.64756 | 45.123 | 1413.78 | 48.384 | 1166.93 | 27,270 |
| 6 | 12.89 | 13.72 | 16.30 | 1.13005 | 14.424 | 263.54 | 16.168 | 171.69 | 18,180 |
| 7 | 24.70 | 42.50 | 44.20 | 1.76538 | 25.037 | 1114.35 | 25.212 | 1049.75 | 7,950 |

| | | | | | | | | | |
|----|--------|-------|-------|---------|---------|---------|---------|---------|--------|
| 8 | 160.99 | 22.70 | 38.15 | 0.24481 | 155.833 | 8215.07 | 215.336 | 3654.47 | 2,270 |
| 9 | 64.97 | 43.13 | 61.35 | 1.40201 | 43.759 | 3558.51 | 58.003 | 2802.16 | 7,950 |
| 10 | 8.89 | 11.98 | 21.65 | 1.77963 | 12.165 | 344.74 | 15.923 | 106.50 | 7,950 |
| 11 | 13.54 | 18.58 | 20.10 | 1.42570 | 14.098 | 294.79 | 14.666 | 251.57 | 25,450 |
| 12 | 71.36 | 36.59 | 38.82 | 0.53907 | 72.014 | 2800.05 | 72.129 | 2611.06 | 3,410 |
| 13 | 30.00 | 42.00 | 49.20 | 1.80400 | 27.273 | 1710.00 | 34.756 | 1260.00 | 2,270 |
| 14 | 56.73 | 34.01 | 42.10 | 0.90296 | 46.625 | 2145.07 | 50.952 | 1929.39 | 11,360 |
| 15 | 23.07 | 47.59 | 49.85 | 2.08454 | 23.914 | 1333.39 | 26.748 | 1097.90 | 11,360 |
| 16 | 22.55 | 21.27 | 23.13 | 0.95913 | 24.116 | 697.37 | 30.150 | 479.64 | 7,950 |
| 17 | 14.96 | 33.48 | 35.41 | 2.26125 | 15.660 | 660.88 | 18.664 | 500.86 | 18,180 |
| 18 | 13.96 | 37.95 | 39.49 | 2.75737 | 14.322 | 590.77 | 14.960 | 529.78 | 450 |
| 19 | 14.23 | 26.60 | 29.03 | 1.93924 | 14.970 | 462.94 | 15.947 | 378.52 | 7,950 |
| 20 | 57.00 | 26.00 | 27.00 | 0.48114 | 56.117 | 1522.00 | 56.370 | 1482.00 | 12,270 |
| 21 | 21.76 | 32.60 | 35.39 | 1.55825 | 22.711 | 838.92 | 23.705 | 709.38 | 510 |
| 22 | 21.77 | 18.45 | 21.43 | 0.90583 | 23.658 | 553.91 | 25.847 | 401.66 | 15,910 |
| 23 | 20.57 | 18.95 | 22.85 | 0.98694 | 23.152 | 621.31 | 27.191 | 389.80 | 18,180 |
| 24 | 34.97 | 30.66 | 32.10 | 0.89852 | 35.725 | 1167.45 | 36.369 | 1072.18 | 20,450 |
| 25 | 34.00 | 24.81 | 26.91 | 0.74169 | 36.282 | 1211.54 | 45.022 | 843.54 | 10,450 |
| 26 | 10.44 | 8.53 | 18.00 | 0.98102 | 18.348 | 637.00 | 35.389 | 89.05 | 10,910 |
| 27 | 28.00 | 19.81 | 22.86 | 0.72783 | 31.409 | 1910.80 | 83.587 | 554.68 | 13,640 |
| 28 | 30.46 | 31.43 | 35.03 | 1.06020 | 33.041 | 2445.88 | 69.822 | 957.36 | 18,180 |
| 29 | 30.79 | 24.61 | 27.39 | 0.80967 | 33.829 | 1798.25 | 605.654 | 757.74 | 450 |
| 30 | 25.70 | 34.70 | 36.80 | 1.36471 | 26.965 | 1195.66 | 32.491 | 891.79 | 15,910 |
| 31 | 11.93 | 16.90 | 19.92 | 1.50167 | 13.265 | 308.83 | 15.503 | 201.62 | 3,640 |
| 32 | 32.13 | 34.09 | 34.73 | 1.06439 | 32.629 | 1216.05 | 35.014 | 1095.31 | 9,090 |
| 33 | 13.75 | 14.39 | 15.70 | 1.07878 | 14.554 | 251.10 | 15.994 | 197.86 | 3,640 |
| 34 | 25.60 | 26.25 | 28.00 | 1.05273 | 26.597 | 784.00 | 28.000 | 672.00 | 11,360 |
| 35 | 27.12 | 20.64 | 22.58 | 0.79726 | 28.322 | 663.74 | 29.395 | 559.76 | 3,640 |
| 36 | 21.98 | 18.93 | 19.55 | 1.44242 | 13.554 | 274.89 | 14.061 | 416.08 | 2,270 |
| 37 | 20.13 | 34.19 | 44.31 | 1.92860 | 22.975 | 1134.09 | 25.594 | 688.24 | 3,640 |
| 38 | 32.61 | 32.92 | 33.82 | 1.02776 | 32.907 | 1117.89 | 33.054 | 1073.52 | 9,900 |
| 39 | 28.37 | 28.29 | 29.05 | 1.00606 | 28.875 | 867.64 | 29.867 | 802.59 | 11,910 |
| 40 | 18.31 | 21.23 | 21.67 | 1.16711 | 18.567 | 414.07 | 19.108 | 388.72 | 22,730 |
| 41 | 20.67 | 40.38 | 43.46 | 1.95987 | 22.175 | 2337.41 | 53.783 | 834.65 | 13,640 |
| 42 | 9.66 | 18.37 | 21.26 | 1.96950 | 10.795 | 300.57 | 14.138 | 177.45 | 18,180 |

Geoelectric Parametric Study using Correlation Analysis

Correlation analysis is the simplest statistical method that reveal the degree of closeness with which two variables X and Y are related to each other. Here Y denotes the 42 sets of well yields Q, and X refers to the corresponding 42 values of any one geoelectric parameter such as aquifer resistivity ρ_t .

The correlation coefficient is given by (Davis, 1986) :

$$r_{xy} = \frac{COV_{xy}}{\sigma_x \sigma_y} \quad (6)$$

where σ_x is the standard deviation of 42 'X' variables and σ_y is the standard deviation 42 'Y' variables and the covariance COV_{xy} is defined by :

$$COV_{xy} = \frac{n \sum_{i=1}^n X_i Y_i - \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n(n-1)} \quad (7)$$

in which $n = 42$ is the sample size and X_i and Y_i are the i^{th} variables.

All the 8×42 geoelectric parameters as X variables and the 42 values of well yields as Y variables were input to the MINITAB statistical software package. The output consisting of 8 correlation coefficients are shown in Table 2. From the values of 8 correlation coefficients (Table 2) it can be noted that the maximum numerical value of r_{xy} is -0.153 against the parameter depth to basement (H). Therefore there is a total lack of correlation between Q and any of the parameter selected for this purpose. Since the correlation coefficients are very poor there is no necessity to apply statistical tests of significance for their validity. Hence cluster analysis is attempted in order to identify groups of bore well sites which have very similar geoelectrical characteristics.

Table 2 - Correlation Coefficients between well yield and Geoelectric Parameters

| Geo=> Aqu- electric Para- meters | Aquifer Resis- tivity, ohm-m (ρ_t) | Aquifer Thick- ness, m (h) | Depth to Base- ment, m (H) | Longi- tudinal Conduc- tance, mhos (L_c) | Avg. Longi- dinal Resis- tivity, ohm-m (ρ_L) | Trans- verse Resis- tance, ohm-m ² (T_r) | Avg. Trans- verse Resis- tivity, ohm-m (ρ_T) | Aquifer Resist- ance, ohm-m ² (A_r) | Yield of the Well, Litres per Hour (Q) |
|---|---|--|---|---|---|--|---|--|--|
| Correlation- Coefficient | -0.131 | -0.091 | -0.153 | -0.139 | -0.116 | -0.143 | -0.10 | 0.068 | |

Geoelectric Parametric Study using Cluster Analysis

Cluster analysis is a multivariate statistical procedure capable of classifying a set of given data into clusters or groups of highly similar entities. Cluster analysis is performed using the multi-variate statistical software package called SYSTAT. All the 336 elements or objects of the 8×42 data matrix of geoelectric parameters (Table 1) are fed as input to SYSTAT. The programme first transforms the data matrix into what is called a distance matrix. The distance used in SYSTAT is the Euclidean distance which is defined as

$$ED_{ij} = \sum_{i=1}^n (X_{ik} - X_{jk})^2 \quad (8)$$

where ED_{ij} is the Euclidean distance between any one object in the i^{th} column and any other

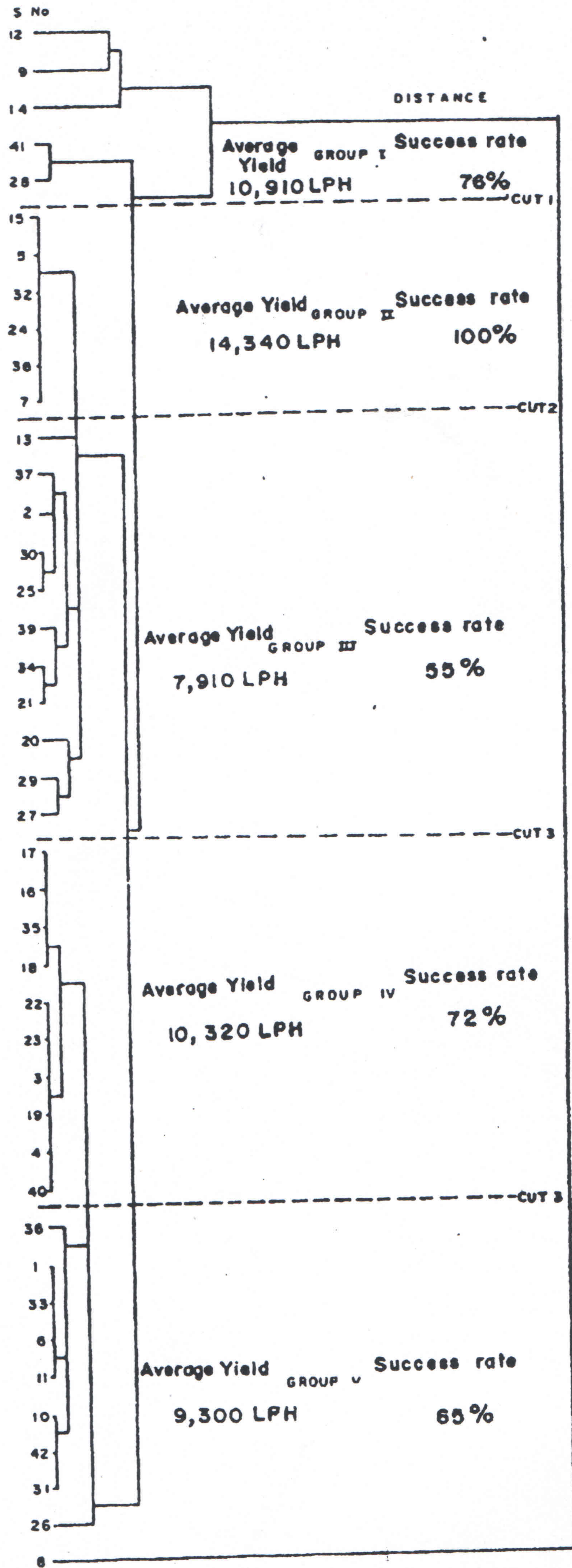


Fig. 2 - Dendrogram of the Geoelectrical Parameters

object in the j^{th} column. n represents the total number of the elements in each column. X_{ik} is the k^{th} element in the i^{th} column and X_{jk} is the k^{th} element in the j^{th} column. Two elements of a data matrix are said to be identical if each object is described by variables having the same magnitude and then the distance between them is zero. The greater this distance, the more dissimilar they are. The output of the programme is Euclidean distances which are represented by a fully nested tree like configuration called dendrogram (Fig. 2). In the dendrogram, all the 42 well numbers are equally spaced along the vertical axis on the left side and the distances used in clustering the wells and clustering the clusters are represented by horizontal lines to a convenient scale.

Results of Cluster Analysis of Geoelectric Parameters

From Fig. 2 it can be observed that wells are grouped at various levels according to the nearness of the Euclidean distances and formed like a tree structure which can be cut into a certain number of coherent branches. Depending upon the first three levels of clustering, four cuts are made in the present case, to obtain 5 groups as shown in Fig. 2. The well sites in each group have evidently the most similar geoelectrical characteristics among themselves. Group II has 100% success rate with an average well yield of 14,340 LPH followed by group I with 76%, group IV with 72%, group V with 65% and group III with 55% success rates. Average yield of each group and percentage of success is shown in the figure. In the field, it is found that the area covered under each group is more or less contiguous implying that they have similar geoelectric characteristics. The measured and derived VES parameters corresponding to the best group are presented in Table 3. It can be observed from this table that the two parameters that exhibit the most narrow range of variation and hence the highest degree of noticeable similarity are the aquifer resistance and the transverse resistance, which represent products of layer thickness and layer resistivity. The average values of aquifer resistance and transverse resistance for this group is 1092 ohm-m² and 1227 ohm-m² respectively. These values can be used as reference values for siting the successful borewells in this type of terrain. Between these two parameters, aquifer resistance has shown more consistent figures than the transverse resistance within this group. However as the aquifer resistance is simply a product of resistivity and thickness, the value of 1092 ohm-m² can be obtained for various combinations of aquifer resistivity and its thickness. Hence there is a necessity to fix the range of these variables. A close examination of Table 3 reveals that the range of aquifer resistivity is of the order of 23 to 43 ohm m and its thickness is varying between 27 to 48 m for productive aquifers.

Table 3 - VES Parameters of the Best Cluster

| VES Parameters => Sl. No. | I Layer Resistivity, (ρ ₁) ohm-m | I Layer Thickness, (h ₁) metres | II Layer Resistivity, (ρ ₂) ohm-m | II Layer Thickness, (h ₂) metres | III Layer Resistivity, (ρ ₃) ohm-m | Depth to Basement, (H) metres | Longitudinal Conductance, (Lc) mhos | Transverse Resistance, (Tr) ohm-m ² | Aquifer Resistance, (A _r) ohm-m ² | Yield per hour (Q) |
|---------------------------|--|---|---|--|--|-------------------------------|-------------------------------------|--|--|--------------------|
| 5 | 116.4 | 2.1 | 43.06 | 27.1 | 3314.0 | 29.2 | 0.64751 | 1413.7 | 1167 | 27,270 |
| 7 | 38.0 | 1.7 | 24.7 | 42.5 | 99.0 | 44.2 | 1.76538 | 1114.4 | 1050 | 7,950 |
| 15 | 104.2 | 2.3 | 23.0 | 47.6 | 1006.5 | 49.9 | 2.08454 | 1333.4 | 1098 | 11,360 |

| | | | | | | | | | | |
|----|------|-----|------|-------|--------|------|---------|--------|------|--------|
| 24 | 66.2 | 1.4 | 34.9 | 30.6 | 1003.9 | 32.1 | 0.89852 | 1167.5 | 1072 | 20,450 |
| 32 | 88.7 | 0.6 | 32.1 | 34.1 | 105.4 | 34.7 | 1.06431 | 1216.5 | 1095 | 9,090 |
| 38 | 49.3 | 0.9 | 32.6 | 32.92 | 931.9 | 33.8 | 1.02776 | 1117.9 | 1074 | 9,900 |

Conclusions

There is no correlation between the yield and any measured or derived geoelectric parameters. However cluster analysis has shown that the areas with an average aquifer resistance of 1092 ohm-m² have successful wells provided the aquifer thickness ranges between 27 to 48 m and resistivity ranges between 23 to 43 ohm.m.

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